FS 2010 Prof. R. Wattenhofer / C. Lenzen

Principles of Distributed Computing
Exercise 8

1 Counting with Asynchronous Wake-up

Recall the counting problem in dynamic networks presented in the lecture. Communication is synchronous, message size arbitrary, and each node has an unique identifier. We want all nodes to learn the number of nodes n.

We assume that the dynamic graph $G = (V, E)$ is 2-interval connected, i.e., for any two subsequent rounds r, $r + 1$, the ("static") graph $(V, E(r) \cap E(r + 1))$ is connected.

Now we drop the assumption from the lecture that all nodes wake up at the same time. Instead, some node $u \in V$ wakes up by itself, while all other nodes start executing the respective algorithm when they receive the first message.

a) Show that it makes no difference if also other nodes may wake up by themselves, i.e., anything you can do if certainly only one node wakes up is still possible.

b) Devise an algorithm that receives an input k and lets u decide whether $k \leq n$ or $k > n$ within $O(k)$ rounds.

Hint: Make u wake up all nodes and collect all identifiers assuming that we have less than k nodes. With a little extra time, one will see more than k identifiers if $n > k$.

c) Use your algorithm as a subroutine for an algorithm that determines n up to a factor 2 in $O(n)$ time. Can n also be determined exactly?

2 Token Dissemination

Suppose node u holds n tokens and a message may carry at most a constant number of tokens. We require that all nodes learn all tokens.

Suppose a token dissemination algorithm exhibits the following "reasonable" behaviour. Nodes decide what to broadcast in round r based on the round number and the set of tokens they know. In particular, once a node knows all tokens, its schedule depends only on the round number. Show that even though the graph is 1-interval connected, it may take $\Omega(n^2)$ time until a correct algorithm (from this restricted class) terminates.

3 Token Dissemination in Well-Connected Graphs*

We change the previous setting in that now the graph is T-interval connected. Show a lower bound of $\Omega(n + n^2/T)$ on the worst-case time complexity of the problem!