

Discrete Event Systems

Exercise 6

1 An Unsolvable Problem

It's the first day of your internship at the software firm Bug Inc., and your boss calls you to his office in order to explain your task for the next three months. He says that many clients complain that the programs of Bug Inc. often contain faulty loops that never terminate. In order to prevent such errors in future, you are asked to implement a program that may check whether a given program will halt on all possible inputs or not.

- a) Try to find a proof that convinces your boss that this is not possible for general programs. Hint: The proof works by contradiction. Assume a procedure $halt(P : Program) : boolean$ that takes a program P and decides whether P halts on all possible inputs or not. Now construct a program X that terminates if $halt(X)$ is false and loops endlessly if $halt(X)$ is true, which yields the desired contradiction.
- b) Your boss still disagrees and proposes the following method: $halt(X)$ simply simulates the execution of program X . If the program terminates it returns true, and if it loops it returns false. Where is the problem of this approach?
- c) Your boss is finally convinced but argues that your proof is a very special case that hardly reflects reality. Are there assumptions under which it is always possible to check whether a program halts or not?

2 Dolce Vita in Rome

In order to relax a little bit from the busy life at ETH, Hector and his girlfriend Rachel decide to spend the weekend in Rome. Besides the cultural attractions, Hector and Rachel are also interested in the great choice of ice cream shops (*gelaterie*) which Rome offers.

During their strolls through Rome, the two students encounter n gelaterie. Assume that these ice cream shops can be ranked uniquely according to their attraction, that is, for any two given shops, Hector and Rachel have a clear preference. For instance, the attraction may be a function of the price of the ice cream, quality, atmosphere of the shop, etc.

Since it's too expensive to eat ice cream on every occasion, the two students apply the following strategy: Whenever a shop i is more attractive than the shops 1 to $i - 1$ which they have encountered so far, they buy an ice cream.

Assume that the ice cream shops appear in a random order, i.e., any one of the first i shops is equally likely to be the best so far. How many ice creams do Hector and Rachel consume during the weekend?