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Catching Elephants with Mice

Sparse Sampling for Monitoring Sensor Networks

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Outline

- Introduction
- VC-Dimension and ε-nets
- Catching Elephants ...
  - ... in Theory
  - ... in Practice
- Simulation Results
- Conclusion & Discussion

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Introduction

- Sensor Networks
  - Ideally: tiny, inexpensive, allowing real-time and fine-grained monitoring
  - Applications
    - Mostly surveillance or environmental monitoring
    - e.g. tracking pollution level in a habitat
- Issues
  - Local and temporal variations
  - Natural faults, adversarial attacks

Introduction – Our Goal

- Detect significant events
  - Monitoring only a small subset of all nodes
  - Using a scheme that scales relatively
- Estimate the size of these events
- Terminology
  - Elephants: "large" events, defined by what fraction ε of the network is affected
  - Mice: the monitoring set we use to detect ("catch") these elephants

Introduction – Assumptions

- No low level issues
  - Reliable communication from nodes to base station
  - Idealized sensing
- Base station knows locations of all sensors
  - But no assumptions over the distribution
- At most one event at any time
- Event-geometry can be described in "nice" ways
  - → Vapnik-Chervonenkis dimension

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VC-Dimension

- Why is a circle simpler than a rectangle?
- (X, R) is called a range space
  - X is a ground set
  - R is a set of subsets (ranges) of X
- Definitions
  - **A $\subseteq$ X is shattered by R** if all possible subsets of A can be obtained by intersecting A with an r $\in$ R
  - The **VC-Dimension of** (**X, R**) is the cardinality of the largest set A that can be shattered by R

VC-Dimension

- Range spaces with infinite VC dimension
  - e.g. if R is the set of all convex polygons
- Relevance for our scheme
  - We will see that we can choose a set of $O\left(\frac{d \log d}{\varepsilon} \log \frac{d \log d}{\varepsilon}\right)$ mice for our scheme to work
  - Thus for events that can be approximated by simple geometric shapes of **constant** VC-dimension the sample size is $O\left(\frac{1}{\varepsilon} \log \frac{1}{\varepsilon}\right)$

ε-nets

- Definition
  - B $\subseteq$ X is an ε-net for X if an event r $\in$ R that affects $\geq \varepsilon|X|$ nodes also affects B (i.e. r $\cap$ B $\neq \emptyset$)
- Construction
  - If we choose $m \geq \max\left(\frac{8d}{\varepsilon}\log\frac{8d}{\varepsilon}, \frac{4}{\varepsilon}\log\frac{2}{\delta}\right)$ nodes from the network at random, we will, with probability of $(1-\delta)$, have an ε-net
  - ➔ $O\left(\frac{d}{\varepsilon}\log\frac{d}{\varepsilon}\right)$ nodes

ε-nets

- Looking at the definition, ε-nets seem to be just the right tool for our problem
- Two major drawbacks if applied directly
  - False alarms
  - No size estimation
- Using ε-nets in another way, we can remedy both problems without increasing the asymptotic size

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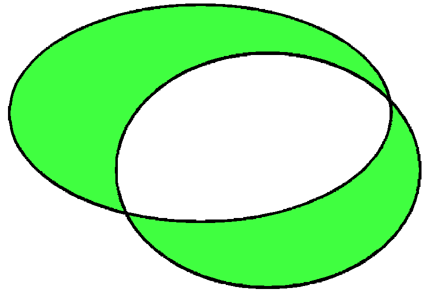
Catching Elephants in Theory



- The symmetric difference
  - $D_1 \oplus D_2 := (D_1 \cup D_2) \setminus (D_1 \cap D_2)$
- If (X, R) has VC dimension d
  - $R' := \{r_1 \oplus r_2 \mid r_1, r_2 \in R\}$
  - Then (X, R') has VC dimension $d' := O(d \log d)$
- Now we can use the following algorithm
  - S: the set of all sensors in our network
  - d: the maximum VC dimension of the elephants
  - ε: the fraction that defines if an event is an elephant

Catching Elephants in Theory - Algorithm

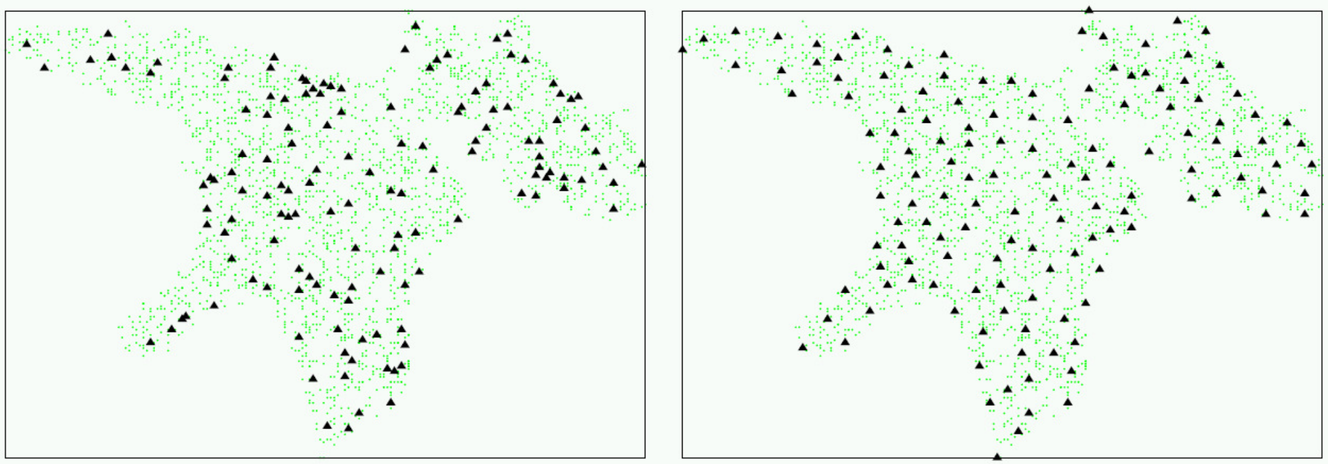
- CatchElephants(S, d, ε)
  1. d' $:=$ O(d log d)
  2. Construct $\varepsilon/4$-net M for S (the mice)
     - w.r.t. the symmetric difference ranges of dimension d'
  3. Let T $\subseteq$ M be the "dead mice"
  4. Compute a disk D
     - containing T and excluding M \ T
  5. K $:=$ |S $\cap$ D| sensors lie inside D
     - If K $\geq 3\varepsilon n / 4$, then the event is an elephant of size K
     - Otherwise the event is not an elephant

Catching Elephants in Theory - Essence

- Constructing the special $\varepsilon/4$-net
  - The number of nodes in $\mathrm{D} \oplus \mathrm{D}^*$ is at most $\varepsilon \mathrm{n}/4$
  - ➔ Size approximation error of $\pm\ \varepsilon \mathrm{n}/4$
- Checking if K $\geq 3\varepsilon \mathrm{n}\ /\ 4$
  - Two-sided guarantee
    - **Every** elephant is reported
    - The algorithm **never** reports events of size $\leq \varepsilon \mathrm{n}/2$
  - False alarms only for events of size $(\varepsilon \mathrm{n}/2, \varepsilon \mathrm{n})$
    - This is the approximation gray zone

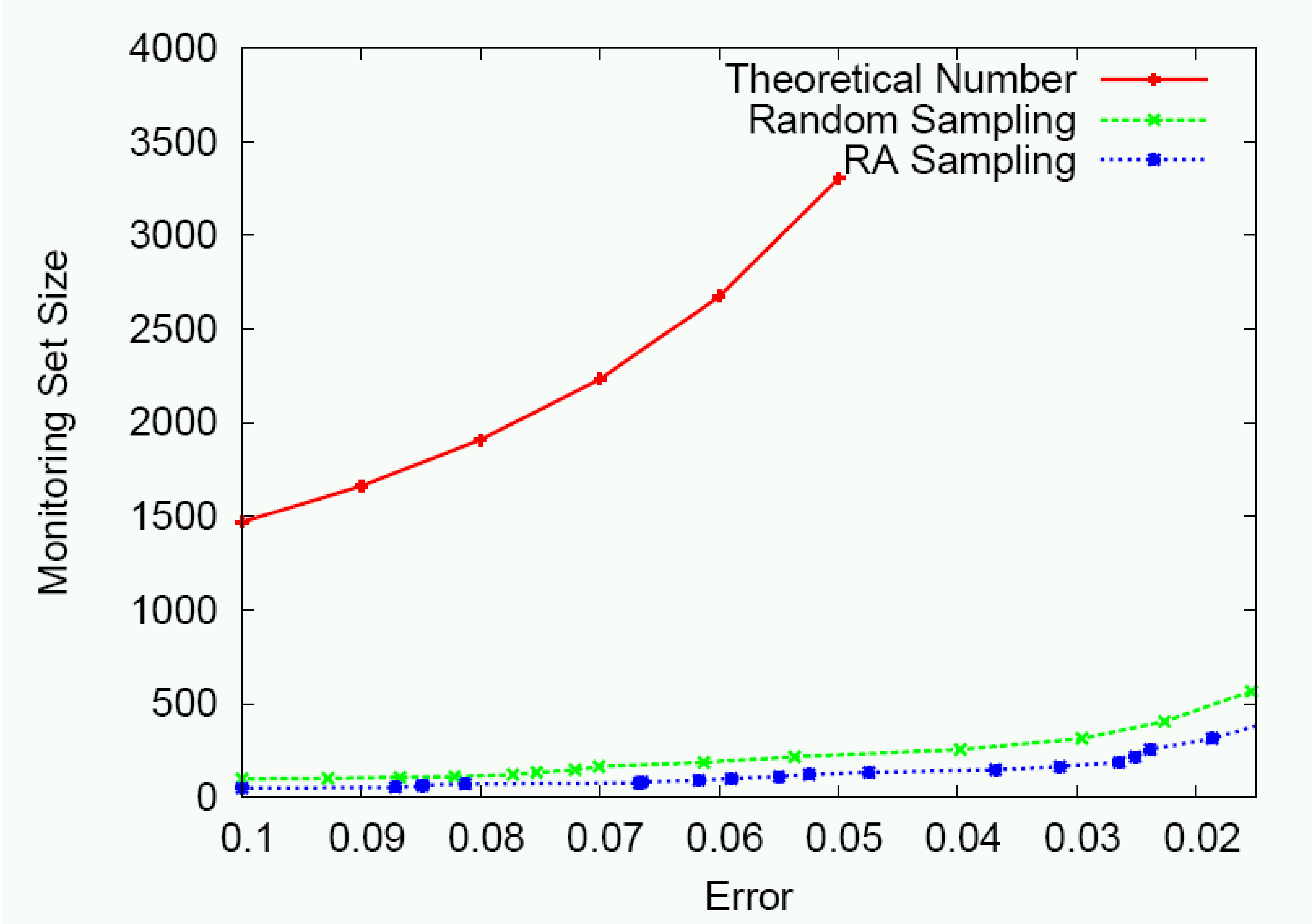
Catching Elephants in Practice

- Estimating Theoretical Pessimism
  - Determine empirically what's "good enough" in practice
- Redundancy-aware Sampling
  - Choose the mice not too close to eachother



Lake (2500 nodes)

Catching Elephants in Practice



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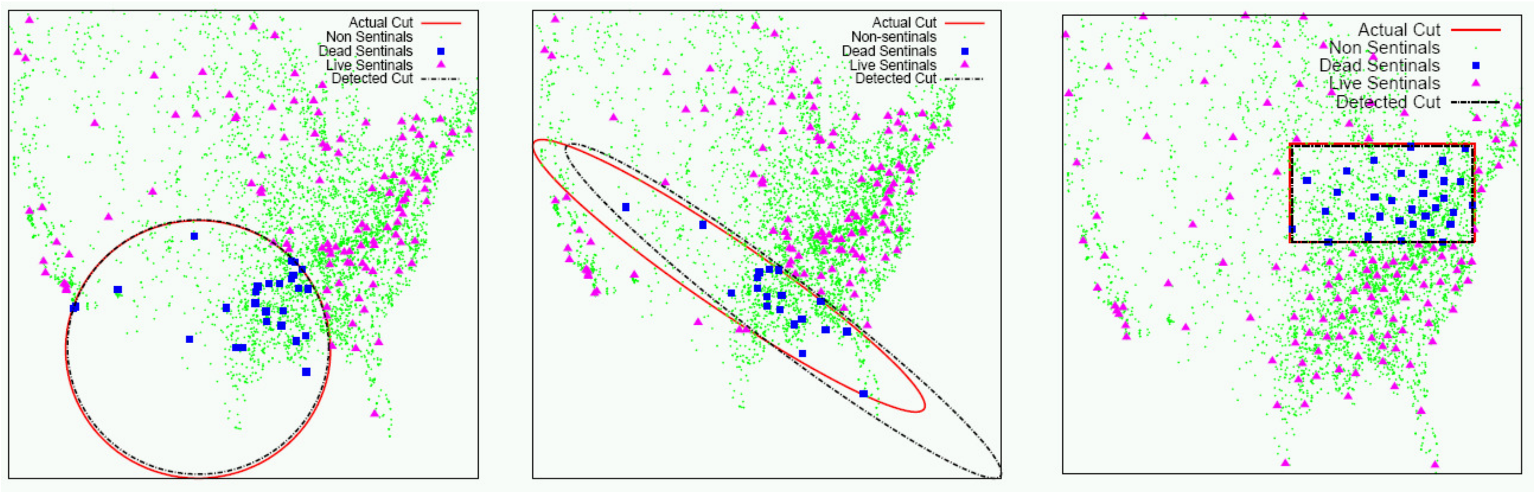
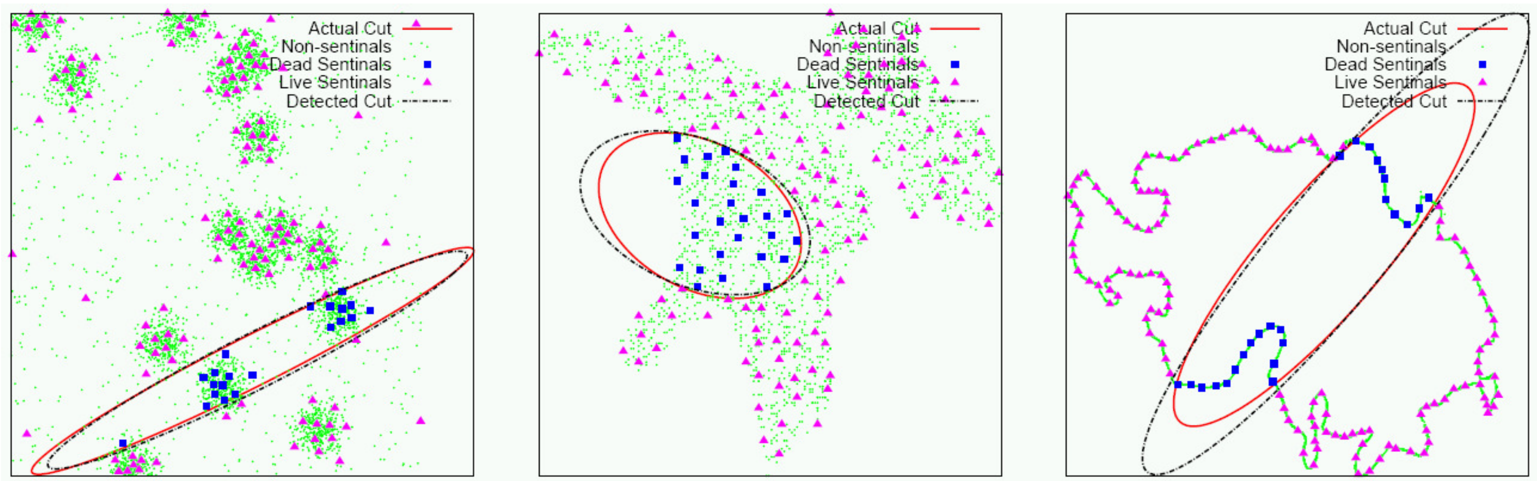
Simulation Results

- Parameters
  - 5 different networks with $n \in [1'000, 45'000]$
  - Geometries: circles, ellipses, axis-aligned rectangles
  - Event-size chosen randomly in $[0.1n, 0.3n]$
- 2000 events for each of the 15 pairs of datasets and event geometries ➔ 30'000 tests
- Monitoring sets constructed by using Redundancy-aware sampling

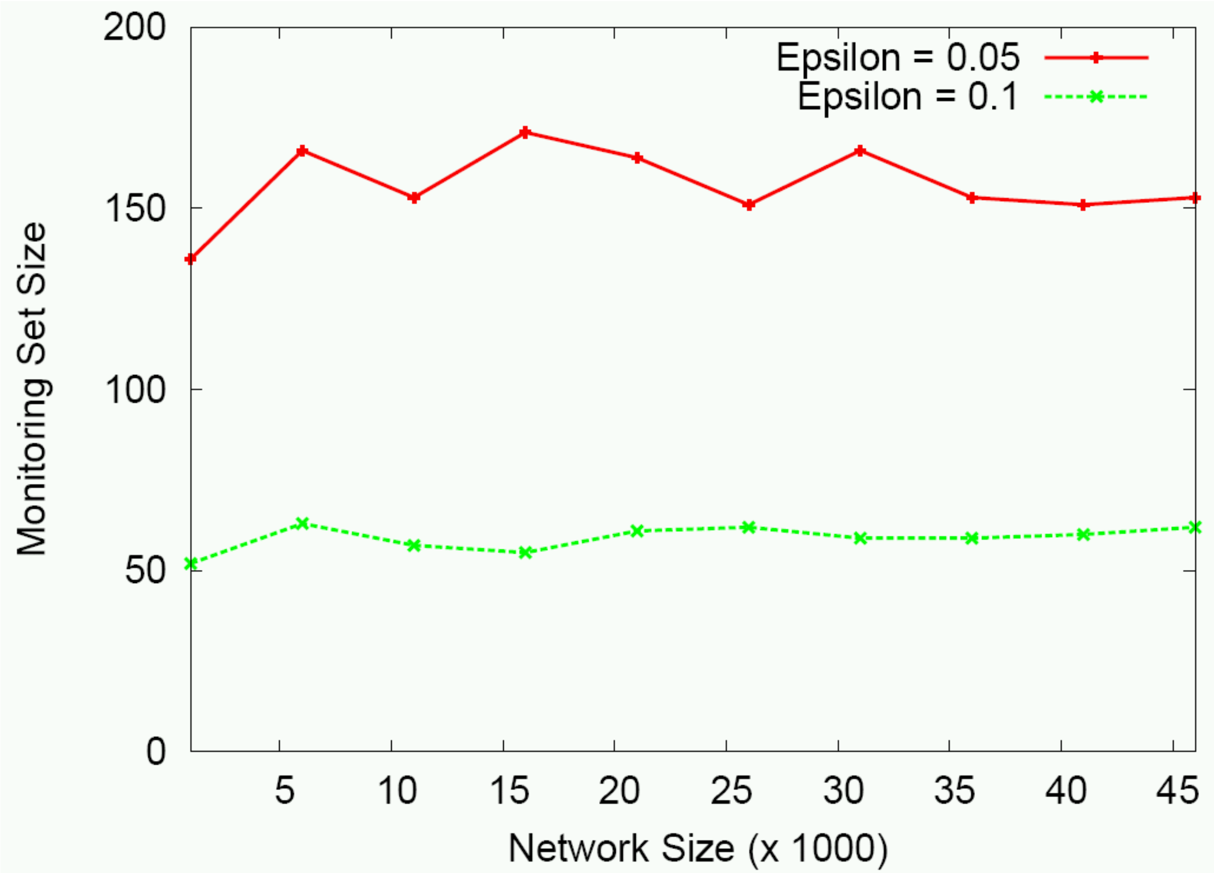
Simulation Results





Simulation Results

- Number of mice vs. total number of nodes



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Conclusion

- The idea of using a random sample is not particularly novel
- Core contributions
  - Quantifying the sample size, using the VC dimension
  - Bridging the gap between theory and practice
- Key idea of the scheme
  - Using ε-nets w.r.t. symmetric difference ranges
  - ➔ two-sided guarantee and size estimation

Discussion – Personal Thoughts

- Interesting ideas
- Sometimes confusing
  - Mixing of asymptotic and plain terms
  - Some (core) details not explained too thoroughly
- Practical focus is notable
  - Why no simulations with polygons?
  - "Quantifying": Most terms are asymptotic
  - "Bridging the gap": Maybe the bridge was built from the theory side

Discussion – Your Turn

- Comprehension questions
- Your opinion
  - Do you think this results are useful?
  - Or do you see difficulties in practical applications?

- How would you extend the scheme for a network with nodes of different importance?