Network Decompositions

Exercise 1: Explain how given a \((C, D)\) network decomposition of graph \(G\), a maximal independent set can be computed in \(O(CD)\) rounds.

Exercise 2: We here see that the \((O(\log n), O(\log n))\) network decomposition that we discussed in the class has the nearly best possible parameters. In particular, it is known that there are \(n\)-node graphs that have girth\(^1\) \(\Omega(\log n / \log \log n)\) and chromatic number \(\Omega(\log n)\) [AS04, Erd59]. Use this fact to argue that on these graphs, an \((o(\log n), o(\log n / \log \log n))\) network decomposition does not exist.

Exercise 3: Given an \(n\)-node undirected graph \(G = (V, E)\), we define a \(d(n)\)-diameter ordering of \(G\) to be a one-to-one labeling \(f : V \rightarrow \{1, 2, \ldots, n\}\) of vertices such that for any path \(P = v_1, v_2, \ldots, v_p\) on which the labels \(f(v_i)\) are monotonically increasing, any two nodes \(v_i, v_j \in P\) have \(\text{dist}_G(v_i, v_j) \leq d(n)\).

Use the existence of \((O(\log n), O(\log n))\) network decompositions, proved in the class, to argue that each \(n\)-node graph has an \(O(\log^2 n)\)-diameter ordering.

References


\(^1\)Recall that the girth of a graph is the length of its shortest cycle.