How Optimal are Wireless Scheduling Protocols?

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Scheduling in Wireless Networks

Scheduling is crucial in wireless networks.

- Modeling *interference* in a typical textbook or algorithms paper:

- A lot of existing work on the impact of interference
- But, are the *foundations* (possibilities, limitations) of MAC layers really understood…?
- Do we have *competitive scheduling* (MAC layer) protocols?
A Simple Scheduling Problem

Consider the following scheduling task $\Lambda$

$\Lambda$: set of communication requests $\lambda_i = (s_i, r_i)$

sender $s_i$, receiver $r_i$
coordinates in Euclidean plane

How many time-slots are required so every request is scheduled?

Compare to capacity of wireless networks...

„The Scheduling Complexity in Wireless Networks“
An example:

<table>
<thead>
<tr>
<th>Time-Slot</th>
<th>Senders:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$:</td>
<td>$v_1, v_4, v_7$</td>
</tr>
<tr>
<td>$t_2$:</td>
<td>$v_2, v_3, v_6$</td>
</tr>
<tr>
<td>$t_3$:</td>
<td>$v_5, v_8$</td>
</tr>
</tbody>
</table>

This scheme uses 3 time slots!

⇒ Scheduling complexity of $\Lambda$ is 3 in this example.

What is the scheduling complexity of wireless networks as a function of $n$? (scaling laws)
Physical Model

- Let us look at the signal-to-noise-plus-interference (SINR) ratio!
- Message arrives if SINR is larger than $\beta$ at receiver

\[ \frac{P_u}{d(u,v)^\alpha} + \sum_{w \in V \setminus \{u\}} \frac{P_w}{d(w,v)^\alpha} \geq \beta \]

- Power level of sender $u$
- Received signal power from all other nodes (=interference)
- Noise
- Path-loss exponent
- Received signal power from sender
- Minimum signal-to-interference ratio
- Distance between two nodes
Related Work

- There is a lot of related work on scheduling
  → numerous practical scheduling protocols
  → wireless MAC layer protocols

- **Capacity** of wireless networks [Gupta, Kumar, Trans.Inf.Theory’00]

- Combined power assignment and scheduling problems
  [Behzad, Rubin, Infocom’05], [Jain, Padhye, Padmanabhan, Qiu, Mobicom’03],
  [Bjorklund, Varbrand, Yuan, Infocom’03], etc…

- Specifically **SINR based scheduling protocols**
  [Ephremides, Truong, Trans.Comm’90], [ElBatt, Ephremides, Infocom’02],
  [Cruz, Santhanam, Infocom’03], etc…

- Comparison between graph-based and SINR-based scheduling
  [Gronkvist, Hansson, Mobihoc’01], etc…

**Competitive scheduling protocol...?**
**Scheduling complexity in wireless networks...?**

Yvonne-Anne Oswald, ETH Zürich @ Infocom 2007
The Scheduling Complexity of Wireless Networks

- n nodes in 2D Euclidean plane
  - Nodes can choose power levels
  - Message successfully received if SINR at receiver sufficient

**Scheduling Complexity \( S(\Lambda) \)**

The minimal number of time-slots required until every request is successfully transmitted (in every network)!

Clearly, \( S(\Lambda) \leq n \)

- Graph-based models...?
- SINR: Fixed power assignment...?
- SINR: Free power assignment...?

- Coloring, independent sets...
- Mathematical optimization

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Example: Graph-based vs. Physical Model

A sends to D, B sends to C

Assume a single frequency (and no fancy decoding techniques!)

Is spatial reuse possible?

NO

Graph-based Models

YES

Physical Model

In Reality!

Let $\alpha = 3$, $\beta = 3$, and $N = 10\text{nW}$
Transmission powers: $P_B = -15\text{ dBm}$ and $P_A = 1\text{ dBm}$

SINR of A at D:

\[ \frac{1.26mW/(7m)^3}{0.01\mu W + 31.6\mu W/(3m)^3} \approx 3.11 \geq \beta \]

SINR of B at C:

\[ \frac{31.6\mu W/(1m)^3}{0.01\mu W + 1.26mW/(5m)^3} \approx 3.13 \geq \beta \]

[Moscibroda, Wattenhofer, Weber, Hotnets'06]
Worst-Case Complexity

- Previous work: heuristics or very strong assumptions on node deployment, topologies
  - randomly, uniformly distributed nodes
  - nodes placed on a grid
  - etc...

What do real networks look like?
\( \rho \) - Disturbance

- Measure for intrinsic difficulty of scheduling task

\[ \chi_\rho := \max_{\lambda_i \in \Lambda} \chi_\rho(\lambda_i), \]

where \( \chi_\rho(\lambda_i) = \max(|\{r_j \mid d(r_j, r_i) \leq d_i/\rho\}|, |\{s_j \mid d(s_j, s_i) \leq d_i/\rho\}|). \]
Power Assignment Policies

Commonly employed/assumed power assignment policies

• All nodes have uniform power
  – Implicit assumption in unit disk graph

• Powers are according to
  – This linear power assignment often (implicitly) assumed
    (e.g., energy metric, topology control, etc... )

• Any kind of fixed power scheme
  – power as a function of link length
  – A discrete set of power levels

Schedule of length $\Omega(n)$ even in scenarios with low $\rho$-disturbance [Moscibroda et al., Infocom 06]
Link Removal Algorithms

postpone links according to some condition until $\text{SINR} > \beta$ for remaining links

- **SRA** (Stepwise Removal Algorithm), [Zander, 92]: remove link with largest row or column sum of inverted gain matrix

- **SMIRA** (Stepwise Maximum Interference Removal Algorithm), [Lee et al., 95], exclude links causing or receiving most interference under optimal power assignment

- **WCRP** [Wang et al., 05](distributed) remove links with $\text{MIMSR} > \zeta$

  $$\text{MIMSR}(i) = \max_{j \neq i} \left\{ \frac{\beta G(i,j)}{G(i,i)} \right\}$$

- **LISRA** (Limited Information Stepwise Removal Algorithm), [Zander, 92], postpone link with the lowest $\text{SINR}$ if equal sending power
Worst Case Complexity

- Schedule length in $\Omega(n)$ for SRA, SMIRA, WCRP, LISRA

- Schedule sets \(\{\lambda_t, \lambda_{t+\log n}, \lambda_{t+2\log n}, \ldots\}\), log $n$ slots necessary
For link $\lambda_i = (-2^i, 2^i)$, the $\tau(i)^{th}$ longest link: $P(s_i) = (2n)^{\tau(i)} 2^{\alpha(i+1)}$

$\Rightarrow$ Short links send with more power than necessary to reach receiver
LDS- Protocol

- Novel power assignment
- New spatial reuse criteria

3 parts:

a) Pre-processing phase
b) Main scheduling loop
c) Subroutine allowed

**Pre-processing phase:**
1: $\gamma_{\text{cur}} := 1; \quad \gamma_{\text{cur}} := 1; \quad \text{last} := d_1$
2: Consider all requests $\lambda_i \in \Lambda$ in decreasing order of $d_i$
3: for each $\lambda_i \in \Lambda$ do
4: if last$/d_i \geq 2$ then
5: if $\gamma_{\text{cur}} < \lceil \log(3n\beta) + \rho \log \alpha \rceil$ then
6: $\gamma_{\text{cur}} := \gamma_{\text{cur}} + 1$
7: else
8: $\gamma_{\text{cur}} := 1; \quad \gamma_{\text{cur}} := \gamma_{\text{cur}} + 1$
9: end
10: last := $d_i$
11: end
12: $\gamma(i) := \gamma_{\text{cur}}; \quad \tau(i) := \tau_{\text{cur}}$
13: end

**Main scheduling-loop:**
1: Define constant $\nu$ such that $\nu := 4N$
2: $t := 1$
3: for $k = 1$ to $\lceil \log(3n\beta) + \rho \log \alpha \rceil$ do
4: Let $\mathcal{F}_k$ be the set of all requests $\lambda_i$ with $\gamma(i) = k$
5: while not all requests in $\mathcal{F}_k$ have been scheduled do
6: $L_t := \emptyset$
7: Consider all $\lambda_i \in \mathcal{F}_k$ in decreasing order of $d_i$
8: if allowed($\lambda_i$, $L_t$) then
9: $L_t := L_t \cup \{\lambda_i\}; \quad \mathcal{F}_k := \mathcal{F}_k \setminus \{\lambda_i\}$
10: end if
11: Schedule all $\lambda_i \in \mathcal{E}_t$ in time slot $t$, assigning $s_i$ a transmission power of $P_t = \nu \cdot d_i^2 \cdot (3n\beta \rho^{\alpha})^{\gamma(i)}$
12: $t := t + 1$
13: end while
14: end for
Performance of LDS

- Results in correct schedule: all communication requests can be transmitted successfully (due to $\chi_\rho$, the maximal interference can be bounded)

- Length of schedule: $O(\chi_\rho \rho^2 \log n (\log n + \rho))$

Recall Scenario S:

- Schedule length in $\Omega(n)$ for SRA, SMIRA, WCRP, LISRA
- LDS $O(\log_2 n)$
Conclusions

• $\rho$ - disturbance measures intrinsic difficulty of scheduling complexity
• Existing protocols heuristics or only efficient if nodes randomly distributed, no worst case guarantees
• LDS polylogarithmic scheduling complexity for low $\rho$-disturbance

• Better understanding of scheduling in wireless networks, improvements for MAC layer?

• Open question: better algorithms possible? For any $\rho$? Distributed?
That’s it…

THANK YOU!